SPATIOTEMPORAL DYNAMICS OF A STOCHASTIC VLSI ARRAY

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ABSTRACT

In this paper we present an analog VLSI array of hysteretic elements that facilitates the exploitation of known properties of stochastic resonance. We present data from a 7×9 array of locally coupled Schmitt trigger elements implemented in a 2μM n-well process. In particular, we demonstrate stochastic resonance in a single element (uncoupled case) with an improvement in output signal to noise ratio of approximately 40 dB. In a spatially extended system (elements coupled via analog transmission gates), we observe an array enhanced effect by measuring the relative firing times between two cells in the array.

I INTRODUCTION

The term *stochastic resonance* (SR) [1][2] was coined to describe the effect where a system's response to a small driving signal is maximized by an optimal amount of input noise. For example, as the input noise strength is increased the signal-to-noise ratio (SNR) of the output of a stochastic resonator will pass through a maximum. This effect is counterintuitive because the presence of noise is typically kept to a minimum in signal detection applications.

The first experimental demonstration of SR was performed by Fauve and Heslot [3] who observed optimal electronic switching in a Schmitt trigger circuit. In their experiment a small periodic signal was used to modulate the width of the hysteresis curve while noise was applied at the input. Fauve and Heslot were able to maximize the response of the detector by applying an optimal amount of noise to the input. By measuring the output power spectral density (PSD) at the driving frequency they showed that the output SNR of the Schmitt trigger passed through a maximum as the input noise was increased. In our experiment a similar effect is realized by applying both signal and noise to a two dimensional array of hysteretic elements. In this case the hysteresis of each element is fixed. Equation 1 is commonly used to calculate the SNR of the system, where s is the amplitude of the peak in the power spectrum at the signal frequency and n is the noise level measured at the base of the peak.

$$\mathrm{SNR} = 10\log\left(\frac{s-n}{n}\right) \tag{1}$$

Fauve and Heslot demonstrated that the output SNR of a nonlinear detector can be maximized by exploiting SR. However, recent theoretical [4] and experimental [5] results show that the output SNR can be further enhanced in a spatially extended system by coupling the elements. In this paper we demonstrate array enhanced SR in a VLSI implementation of a 2D array of hysteretic elements using nearest neighbor coupling and global signal and noise sources.

II CIRCUIT DESCRIPTION

Our CMOS implementation consists of a 7×9 array of locally coupled hysteretic elements. The input of each element is coupled to the output of four neighbors (left, right, top, bottom) and two global signals. As illustrated in Figure 1, the coupling is implemented with analog transmission gates between the input of the local cell and the outputs of neighboring cells. The strength of coupling is set by applying a bias voltage to the gates of the p-type and n-type MOS transistors in the transmission gate. The width of the hysteresis curve is determined by the gate voltage on transistors M3 and M4. With transistors M3 and M4 biased off, the circuit behavior is similar to that of two inverters. However, when M3 and M4 are biased on, their effect is to pull down node N1 and pull up node N2 respectively during the appropriate phase of the circuit so

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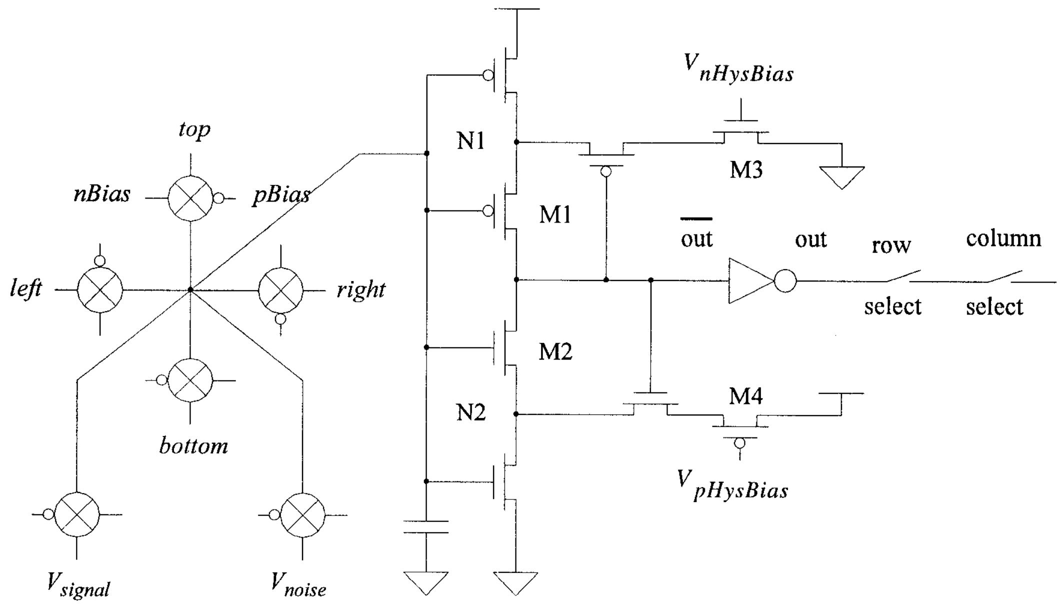

Figure 1 Schematic diagram of a single cell showing hysteretic element and coupling circuitry. Parameters $V_{nHysBias}$ and $V_{pHysBias}$ determine the amount of hysteresis in the Schmitt trigger. The coupling strength between neighboring cells is determined by voltages set on $nBias$ and $pBias$ inputs of analog transmission gates (coupling strengths for global signal and noise sources are similarly set).

that the input-output response exhibits hysteresis. For example, when the output is low ($\overline{out}$ is high), node N2 is slightly pulled up so that the turn-on of transistor M2 is delayed. Direct measurements show that a single element is capable of achieving a hysteretic range varying from 0 Volts to as much as 3 Volts (with Vdd = 5 Volts). Outputs from the array are time-multiplexed onto a single data line by way of two transmission gates at each site (see Figure 1). The control signals for the transmission gates are generated by horizontal and vertical shift registers that propagate a control bit for column and row selection respectively. The output of the array can be viewed on a multisync monitor or can be digitally recorded on a microcomputer. The data presented in this paper was digitally recorded at a sampling rate of 420 Hz

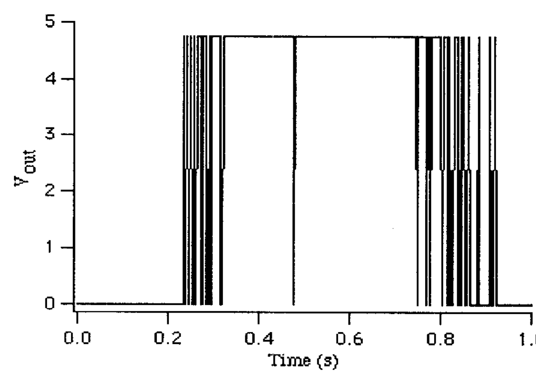

Figure 2(a) Typical voltage output of cell (4,5). V_{signal} = 0.22 Volts (1Hz sine wave). V_{noise} = 0.58 Vrms.

III STOCHASTIC RESONANCE OF A SINGLE CELL

In the first experiment the response of a single element is measured without coupling between cells. This is accomplished by setting the gates of the coupling pMOS transistors to Vdd and the gates of the coupling nMOS transistors to Vss. Figure 2(a) shows a segment of a typical time series of the voltage output of cell (4,5) with V_{signal} = 0.22 Vpp and V_{noise} = 0.58 Vrms. The bias voltages $V_{nHysBias}$ and $V_{pHysBias}$ are set so the signal is completely inside the hysteresis curve. At this setting, the input signal amplitude of 0.22 Vpp is well below the detection level of the Schmitt trigger.

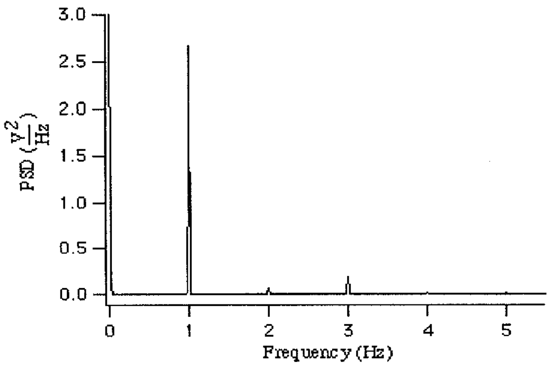

Figure 2(b) The power spectrum of the voltage time series from cell (4,5). The cells are uncoupled.
V_{signal} = 0.22 Vpp (1 Hz). V_{noise} = 0.58 Vrms.

Figure 2(b) shows the power spectrum from the time series (which consists of 66,665 points). The power spectrum shows a strong peak located at the driving frequency with additional peaks at odd multiples of the driving frequency. The occurrence of smaller peaks at even multiples is an indication of a lack of total symmetry in the transfer curve of the Schmitt trigger [6]. The figure also shows the output noise is almost completely suppressed by the system. This phenomenon was also observed by Fauve and Heslot and has since been explained by McNamara and Wiesenfeld [7]. They showed that in SR the total power output of a bistable system is constant; power that is transferred to the signal is subtracted from the total noise power. This effect is illustrated in Figure 3(a) which shows the total power output as a function of input noise strength. Figure 3(b) shows the output SNR for the center cell as a function of input noise strength. The figure shows a maximum in SNR at an optimal noise setting of

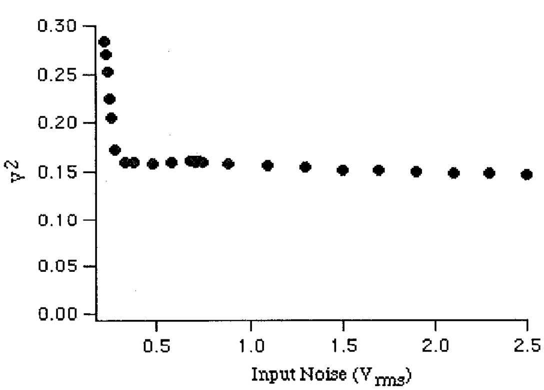

Figure 3(a) The total power output of cell (4,5) as a function of input noise.

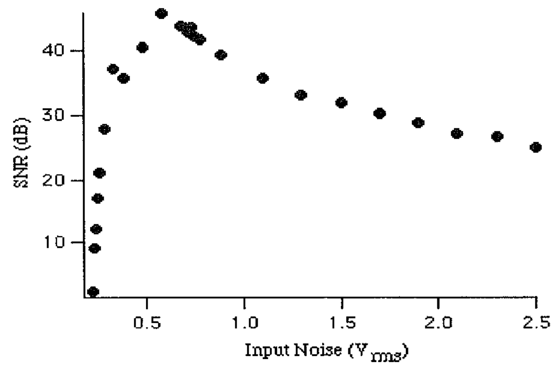

Figure 3(b) SNR as a function of input noise. The response of the trigger to a small periodic signal is maximized by an optimal amount of input noise.
V_{signal} = 0.22 Vpp (1 Hz). V_{noise} = 0.58 Vrms.

V_{noise} = 0.58 Vrms. Note the 40 dB increase in SNR as the noise input is increased from 0 Vrms to 0.58 Vrms.

IV ARRAY ENHANCED STOCHASTIC RESONANCE

In *array enhanced stochastic resonance* (AESR) the response of a detector array is enhanced by cooperative behavior between cells in the array. This cooperation is brought on by coupling between cells which can take on different forms such as in local [4], global [8] and nonlinear coupling schemes [9]. The purpose of this project is to demonstrate AESR in a VLSI array of simple detectors. In this experiment VLSI technology presents itself as a natural solution to an otherwise difficult electronics experiment because the construction of such a system by discreet components would be extremely time consuming. In order to keep our design simple we use a global noise source which prevents a more relevant study of AESR, as in [4], where local noise seems a necessary feature.

Figure 4(a) shows the output SNR of cell (4,5) as a function of input noise and coupling strength. The figure shows little change in the peak output SNR of the cell as the coupling strength is increased. This result is not surprising since we are using a global noise source. Even though the maximal SNR appears independent of the coupling strength, an array enhanced effect can be measured by studying the relative firing times between two cells in the array. This is done by taking the difference in the phase component of the Fourier transforms of two different cell time series. In this case the outputs of cell (4,5) and cell (3,5) were measured. Figure 4(b) shows the phase

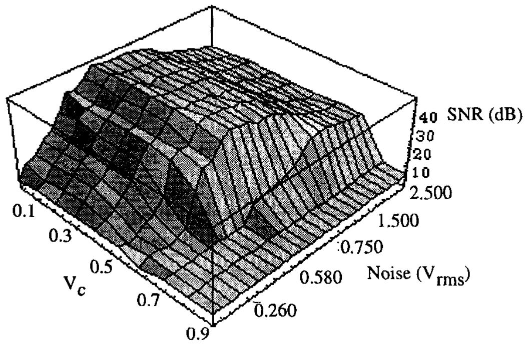

Figure 4(a) Output SNR of cell (4,5) as a function of input noise and coupling strength. The cell's response to a small periodic signal is maximized by an optimal amount of input noise.

difference $\Delta\phi = \phi_{4,5} - \phi_{3,5}$ as a function of noise input strength and coupling strength. It is clear that for an optimal coupling strength ($V_c \approx 0.45$ Volts) the two cells become almost totally synchronized. Figure 4(b) illustrates that maximal synchronization occurs over a range of coupling strengths and that for very strong coupling the cells appear to desynchronize (this effect is currently being investigated). For zero coupling each cell demonstrates the same SNR characteristics but they do so slightly out of phase with each other. As coupling increases the cells start to fire in groups. At an optimal coupling strength the cells synchronize and the array behaves as a single element.

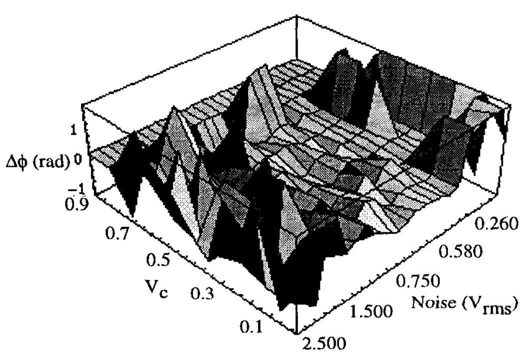

Figure 4(b) The difference in phase response between two cells as a function of input noise and coupling strength. The two sites are nearly phase locked for an optimal coupling strength ($V_c \approx 0.45$ Volts). V_c is applied to the gates of the coupling transistors so that $nBias = V_c$ and $pBias = Vdd - V_c$.

V CONCLUSION

We have shown a strongly enhanced response (40 dB) in a single hysteretic element to a low amplitude signal in the presence of an optimal amount of input noise. We are also able to demonstrate an array enhanced effect by measuring the relative phase responses between two cells in the presence of global noise and increasing coupling strength.

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